Laser frequency noise coupling in LISA

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Summary

Coupling of frequency noise to absolute ranging error, Michelson X, TDI 2.0

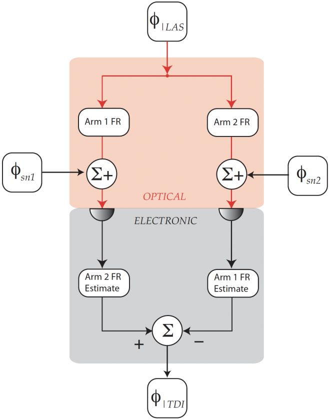
- Displacement-equivalent noise $\tilde{x}(f)$ from fractional laser frequency noise $\tilde{y}(f) = \tilde{\nu}(f)/(2.82 \times 10^{14}\,\mathrm{Hz})$ depends on (differential error in absolute ranging) $= \Delta$

$$\begin{aligned} \tilde{x}(f) &= |\Delta| \cdot \tilde{y}(f) \\ &= (1\,\mathrm{pm}/\sqrt{\mathrm{Hz}}) \cdot \frac{|\Delta|}{2\,\mathrm{m}} \cdot \frac{\tilde{\nu}(f)}{141\,\mathrm{Hz}/\sqrt{\mathrm{Hz}}} \end{aligned}$$

- Coupling factor is
  - Frequency-independent
  - Insensitive to arm length mismatch

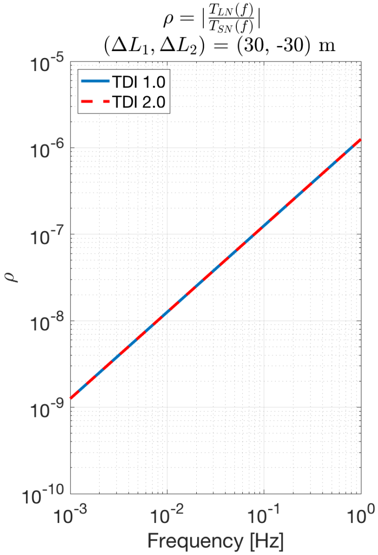
Calculation Framework

Effect of frequency noise in TDI 1.0 and TDI 2.0 derived by McKenzie and Shaddock, JPL Technical Note LIMAS-2008-001 (2009). See also *LISA Frequency Control White Paper*, 2009, available on Atrium as `https://tinyurl.com/fcst-2010`



- Compute separately $T_{\rm LN}$ and $T_{\rm SN}$, transfer function from laser noise and shot noise to TDI output.
- $\rho \equiv |T_{\rm LN}/T_{\rm SN}|$.
- Coupling factor is $R \equiv \tilde{y}/\tilde{x} = \rho c/(2\pi f)$.
- Result from ρ evaluation: $R = |\Delta|$, independent of frequency

Numerical result



- ρ is the same to high precision for TDI 1.0 and TDI 2.0
- $\rho \propto f$.

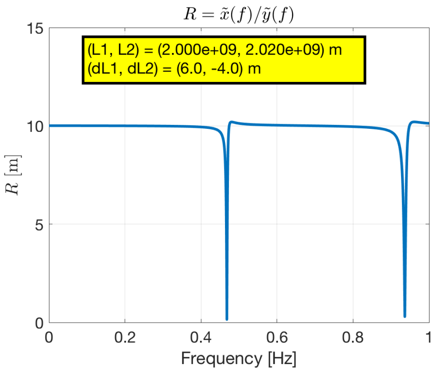
Transfer functions, TDI 1.0 and 2.0

To $\mathcal{O}(d/L)$, $(d, L) =$ (arm length mismatch, nominal arm length) $\approx (0.01, 1) \cdot 2 \times 10^9$ m,

$$
\begin{aligned}
|T_{\mathrm{SN}_{1.0}}|^2 &= 16 \sin^2(\omega L) \\
|T_{\mathrm{SN}_{2.0}}|^2 &= |T_{\mathrm{SN}_{1.0}}|^2 \cdot 4 \sin^2(2\omega L)
\end{aligned}
$$

$$
\begin{aligned}
|T_{\mathrm{LN}_{1.0}}|^2 &= 16 \Delta^2 \omega^2 \sin^2(\omega L) \\
|T_{\mathrm{LN}_{2.0}}|^2 &= |T_{\mathrm{LN}_{1.0}}|^2 \cdot 4 \sin^2(2\omega L)
\end{aligned}
$$

Mismatched arms



- Numerical calculation for 1% arm length mismatch
- Good agreement with analytic result for matched arms except for two nulls
- Nulls don't affect requirement based on coupling factor

Noise from antialias filter: summary

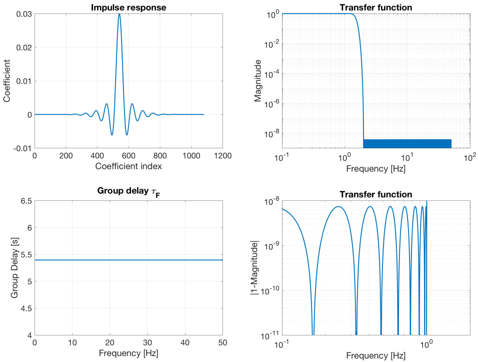
Coupling of antialias filter with laser frequency noise discovered at APC has recently been studied at JPL

- Effect is independent of ranging error
- In passband, FIR filter is nearly perfect delay, $F = e^{-i\omega\tau_F}$, typically $\tau_F = 5\,\mathrm{s}$
- Find for rss arm velocity v: $\Delta_F \equiv \tau_F \cdot v/2 =$ mean armlength change over τ_F,

$$\boxed{\tilde{x}(f) = |\Delta_F|\tilde{y}(f)}$$

  the same as frequency noise from static ranging error Δ_F
- Can be perfectly compensated by applying advance $= \tau_F$ as part of TDI processing
- Applying advance is equivalent to acausal filter using τ_F future data
- After compensation, filter effect is negligible

Filter = Delay



- Antialias (low-pass) FIR filter used in "Experimental Demonstration of Time-Delay Interferometry for the Laser Interferometer Space Antenna," de Vine et. al., PRL, 2010
- Passband frequency response = unity to better than 1×10^{-8} (filter design)
- Group delay constant at all frequencies (property of symmetric FIR filter)

Uncompensated filter effect, APC

- APC draft gives PSD of error from filter

$$S_{X_{2.0}}(\omega) \approx 32 S_p \mathcal{K}_{\mathcal{F}}(\omega) \left(\frac{\omega}{f_s}\right)^2 (\dot{L_2}^2 + \dot{L_3}^2) \sin^2(\omega L) \sin^2(2\omega L)$$

- The filter error enters via first moment, $\sqrt{\mathcal{K}_{\mathcal{F}}} = |\sum_k k\alpha_k|$, $\alpha_k =$ filter coefficients. For symmetric FIR filter, $\mathcal{K}_{\mathcal{F}}(\omega)/f_s^2 = (\tau_F + 1/f_s)^2 \approx \tau_F^2$
- Converting $S_{X_{2.0}}$ to $\tilde{x}(f)$,

$$\tilde{x}(f) = \sqrt{\frac{S_{X_{2.0}}}{|T_{\mathrm{SN}_{2.0}}|^2}} = \sqrt{\frac{S_{X_{2.0}}}{64 \sin^2(\omega L) \sin^2(2\omega L)}} = |\Delta_F| \cdot \tilde{y}(f),$$

where $\Delta_F \equiv v \cdot \tau_F/2$, $v \equiv \sqrt{\dot{L_2}^2 + \dot{L_3}^2}$.